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Is English consequence compact?

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Abstract

By mimicking the standard definition for a formal language, we define what it is for a natural language to be compact. We set out a valid English argument none of whose finite subarguments is valid. We consider one by one objections to the argument's logical validity and then dismiss them. The conclusion is that English—and any other language with the capacity to express the argument—is not compact. This rules out a large class of logics as the correct foundational one, for example any sound and complete logic, and in particular first-order logic. The correct foundational logic is not compact.

KEYWORDS

compactness, first-order logic, logic, logical consequence, philosophy of logic, second-order logic, validity

Logic must underwrite the validity of valid arguments and the invalidity of invalid ones. It must respect facts about which sentences logically follow from others. Graham Priest is surely right to call this employment of logic its 'canonical application'.[1] So much is agreed on all hands. But which logic best carries out the task?

A key metalogical difference between logics such as propositional and first-order logic on the one hand and second-order logic or typical infinitary logics on the other is that the former are compact whereas the latter are not.[2] Recall that if Γ is any set of sentences of a logic, δ any sentence in its language, and $\vDash$ is the logic's consequence relation, we may characterise the logic's compactness as follows:

If $\Gamma \vDash \delta$ then $\Gamma^{\text{fin}} \vDash \delta$ for some finite subset Γ^{fin} of Γ

Now compactness is defined for formal languages equipped with a consequence relation. English, not being a formal language, is not usually considered to be compact or non-compact. But the notion of compactness is easily extended to English and any other natural language. Taking our cue from the formal definition, say that English consequence is compact just when: for any set S of English sentences and any English sentence s, if S entails s then some finite subset of S entails s. Or to put it another way, any valid argument of English has a valid finite subargument.[3] This raises a natural question: is English consequence compact or non-compact?

Our article answers this question. We think it's of the utmost importance for the philosophy of logic—for applied logic, more specifically—that the question be answered clearly and, if at all possible, conclusively. This will be our aim: to show in detail and as decisively as philosophical argumentation allows that English is non-compact.

Before we present the argument, we must explain the notion of a foundational logic. An analogy will be useful. Pure mathematics investigates many different geometries: Euclidean planar, three-dimensional and higher-dimensional; spherical; hyperbolic; and many more. Physicists and applied mathematicians, in contrast, are concerned with a small subset of these: the geometries compatible with physical laws. Indeed, many physicists want to know how physical spacetime is actually structured, not just how it could be structured compatibly with the physical laws. So one can ask which of the many available geometries models that of actual space or spacetime. As has been recognised since Einstein (1921), indeed for decades prior, that is a question for physics rather than mathematics. Similarly, what we might call *pure* logicians investigate a variety of logics. The *applied* logician wants to know what the logic of our language is, and this is what we call the foundational logic. Pure logicians tend to be mathematicians and applied logicians philosophers (but not exclusively so).

Over the first half of the 20th century, first-order logic emerged as logicians' and mathematicians' foundational logic of choice.[4] Although first-order logic has been challenged as the foundational logic, it remains the default such logic in the sense that no candidate has garnered sufficient support to overthrow it and metatheoretic investigations almost always take place in a first-order system (often a first-order set theory) or some informal rendering of it. First-order logic is uncontroversially part of the correct foundational logic, and any extension of it only controversially so. Our argument will show that the foundational logic cannot be compact so in particular cannot be first-order logic.

The interest in the question of English's compactness largely stems from its upshot for our choice of foundational logic: the logic that best underwrites logical consequence in English. We therefore need some understanding of English consequence in order to be clear on the target phenomenon. It is difficult to make a choice here while remaining dialectically neutral. If we assume from the start that English consequence includes instances requiring second-order or infinitary resources or if we assume that certain quantifiers are logical, then it can be a short step to showing that English is non-compact. *We will be careful to presuppose only (relatively) uncontroversial characteristics in our account of English consequence.*

First, when we first introduce students to logic, we say that validity is a matter of the impossibility of true premises and false conclusion. And the thought that validity includes necessary truth-preservation has been a dominant one in the history of writing on logical consequence.[5] Of course, necessary truth-preservation is rarely taken to be identical to English consequence, and the most popular extra ingredient is that validity holds *in virtue of form*. The most important aspect of form is an account of the logical constants: the expressions whose interpretation is held fixed when we determine the form of a sentence. Again, it is difficult to take a position on the logical constants without begging the question in favour of one logic or other. If the logical constants include quantifiers such as ‘there exist uncountably many’, then the non-compactness of English is inevitable. We will not invoke any assumptions about the logical constants here, but be led to conclusions about them by our discussion.

1 | THE ARGUMENT

For ease of reference, we call the following argument, which has appeared in different guises in the literature on compactness, $\mathcal{A}$:

$$\begin{array}{c} \text{There is at least one planet.} \\ \text{There are at least two planets.} \\ \vdots \\ \text{There are at least } n \text{ planets.} \\ \vdots \\ \hline \text{There are infinitely many planets.} \end{array}$$

Here ‘n’ ranges over English numerals: ‘one’, ‘two’, ‘three’, ..., ‘one million’, ‘one million and one’, and so on.[6] Argument $\mathcal{A}$ seems valid, and evidently no finite subset of its premise set entails its conclusion. It seems to follow that English consequence is non-compact. The moral carries over to any natural language into which the argument is translatable. If $\mathcal{A}$ is valid so that natural-language consequence is non-compact, it follows that first-order logic (FOL), which, as is well known, *is* compact, cannot be the correct foundational logic. The same goes for any compact logic. And as is also well known—and a consequence of compactness—$\mathcal{A}$'s conclusion is not formalisable by any sentence of FOL, otherwise $\mathcal{A}$'s first-order formalisation would be a valid FOL-argument none of whose subarguments with finite premise sets is valid.

Other philosophers have argued that an argument along $\mathcal{A}$'s lines witnesses English's non-compactness; see for example Boolos (1975, p. 49) and Oliver and Smiley (2013, p. 238). Champions of second-order logic such as Boolos will of course take $\mathcal{A}$ to be valid, since it is formalisable as a valid second-order argument. Champions of plural logic such as Oliver and Smiley (and Boolos) would likewise also regard $\mathcal{A}$ as valid. Moreover, the literature does not, so far as we know, contain any overt resistance to the idea that an argument such as $\mathcal{A}$ is valid. What these and other champions of second-order or plural logic have in common, however, is that they more or less lay down the validity of natural-language arguments such as $\mathcal{A}$ that are formalisable in second-order logic or plural logic but not first-order logic. They do not justify the claim that these specific arguments are logically valid in a detailed and direct way; their validity falls out only as part of a much broader and more theoretical argument that second-order logic or plural logic is logic.[7]

Although we too take $\mathcal{A}$ to be valid, we believe that this conclusion must be justified with the utmost care and caution, and if possible without invoking controversial theoretical assumptions in the philosophy of logic. That's something that has not been done before, and our aim is precisely to fill this lacuna. To this end, we will work on behalf of our opponents. We set out the most plausible objections to $\mathcal{A}$'s validity we can think of; and then we answer them. We frame the discussion as an objection to first-orderism, defined as the thesis that first-order logic is the correct foundational logic. This is for two reasons: first-order logic is compact, so can stand proxy for any compact logic; and, as indicated, it is a baseline logic, in the sense that no one seriously doubts that *at least* first-order logic is required to capture the validity of English argumentation.[8]

Resistance to $\mathcal{A}$'s validity may take one of four forms. The first is to question whether English has infinitely many sentences. The second is to question whether $\mathcal{A}$ is really *logically* valid or valid only in some weaker sense. The third form of dissent is to maintain that $\mathcal{A}$ can only be understood under some finite description. The fourth and final objection is to query whether there is a determinate notion of English consequence. We take these objections in turn, and rebut each of them. Our conclusion will be that $\mathcal{A}$ *is* valid. This shows that English is not compact; in particular, it sinks first-orderism.

2 | THE FINITUDE OBJECTION

Does English really consist of infinitely many sentences? It has a finite lexicon (vocabulary), it only allows sentences of finite length, and in practice only sentences shorter than some finite length will ever be uttered, written or understood. So one might object that $\mathcal{A}$ is not really an argument of English, since its premise set contains sentences of longer length than any finite bound. In which case, it would follow that English is compact, since the consequence relation of any language with finitely many sentences is trivially compact. As a consequence, the $\mathcal{A}$-based objection to first-orderism runs aground; there is no such argument upon which to base the objection.

Our response to the finitude objection is threefold. The first is that the finitude objection sits ill with the idea that the correct foundational logic is *at least* FOL. If you think there are only finitely many natural-language sentences, a finite fragment of FOL will suffice to model these sentences' implicational structure. The correct logic will then be this finite fragment of FOL, not FOL itself. And it will not do to suppose that a finite fragment of FOL does the job but that it's simpler to pretend that it's FOL for most intents and purposes. It's extremely simple to define restrictions of FOL whose sentences are all shorter than some upper bound.

Second, our best theory of logical consequence should apply not just to current natural language but to its possible extensions. Natural languages accrue words by the day, and those without a written record shed them too. In a few years, English will have acquired sentences it cannot express today, just as the sentence ‘I can't play my DVD on my laptop’ would have been incomprehensible to Victorians. No logician worth her salt is interested in giving an account of logical consequence that is valid merely for a precise moment in time. Furthermore, even if natural languages were set in stone and no longer subject to change, they *could* have been different from what they are. Just as a meteorite crashing on Earth in 1901 might have prevented us from ever adding the words ‘DVD’ and ‘laptop’ to English, so future natural languages could be different from how they in fact turn out. As applied logicians, our interest should be not in any particular natural languages but in their possible extensions. It is scarcely contestable that these include infinitely many sentences. Incontrovertibly, at least some of them can express argument $\mathcal{A}$.

Finally, observe that linguists, philosophers, psychologists and other theorists of language conceive of *actual* natural languages as made up of infinitely many sentences.[9] These sentences are of finite, but arbitrary, length; hence there are infinitely many of them. More precisely, the set of natural-language sentences is usually specified by a set of recursive procedures, which generate sentences of arbitrary length. For example, all the following are sentences of English:

> Your grandparents were tall;
> Your great-grandparents were tall;
> Your great-great-grandparents were tall;
> ...

Now as a matter of empirical fact, there is some finite number N such that you do not have any great^N-grandparents (which N is the least such may be vague). But that does not affect the point that the infinitely many listed sentences are bona fide sentences of English.

In sum, the finitude objection conflicts with a foundational assumption in the study of language. It also sits ill with our starting point of FOL and the philosophical logician's proper task of accounting for logical relations in extensions of English. For these reasons, we find the finitude objection particularly unpromising. Let us turn to a different attempt to block our objection that English is compact based on $\mathcal{A}$'s validity.

3 | THE *MATHEMATICAL* VALIDITY OBJECTION

For argument $\mathcal{A}$ to witness English's non-compactness, it must be a *logically* valid argument. If it is valid only in some weaker sense, then it fails to make the antecedent of compactness true,[10] and so fails as a counterexample. There are of course many kinds of non-logical validity, for example metaphysical validity or mathematical validity. Here is an example of a metaphysically valid argument, on Kripkean assumptions: 'The stuff in the glass is water; *so* the stuff in the glass is H_2O'. This argument is truth-preserving of metaphysical necessity but it is not *logically* valid, since the vocabulary on which its validity depends—'water', 'H_2O'—is not plausibly logical. Similarly, the objection goes, the vocabulary on which the validity of $\mathcal{A}$ depends—'one', 'two', ..., 'infinitely many'—is not properly logical.

Clearly, $\mathcal{A}$ is at least mathematically valid. Failure to appreciate $\mathcal{A}$'s validity is a mathematical failing, in a way that failure to appreciate the validity of 'The stuff in the glass is water; *so* the stuff in the glass is H_2O' is not a mathematical failing.[11] So might $\mathcal{A}$ be merely mathematically valid, and not logically valid?[12]

We take it that 'infinitely many' is the only plausible source of controversy here. For any finite n, that there are at least n many planets is of course first-order definable (formalising 'is a planet' as a predicate letter F), and we take it as uncontroversial that the correct foundational logic is at least as strong as first-order logic. This needs to be first-order logic with identity, but we also take it to be (virtually) uncontroversial that identity is logical. Certainly, all of the most plausible criteria for logical constanthood, such as topic-neutrality and generality, apply to identity. So, if this objection is to get off the ground, it must be that the quantifier 'there are infinitely many' is not a logical constant.

As noted at the start, we must be careful dialectically. Our argument is what we might call 'bottom-up' rather than 'top-down': we do not want to invoke a general criterion of logical constanthood. To put it another way, we want to argue that $\mathcal{A}$ is logically valid on independent grounds and thereby show that 'there are infinitely many' is a logical constant (as we ultimately believe it is), rather than presuppose this fact to show that $\mathcal{A}$ is logically valid. So if we defend a criterion for logical constanthood that 'there are infinitely many' passes, we risk begging the question in favour of English's non-compactness. Further, it would take us far beyond the scope of this paper to provide an account of the logical constants. Fortunately, we do not have to.

$\mathcal{A}$ is mathematically valid, that is, valid assuming the mathematical fact that to be infinite is to be at least as great as 1, at least as great as 2, ..., at least as great as n, ... Our claim is that $\mathcal{A}$ is also logically valid, and the objection is that it is mathematically but not logically valid. To vindicate our claim, we must exploit a well-known fact about mathematical validity, appreciated by logicians. This is the fact that mathematically valid arguments may be turned into logically valid arguments from an appropriately specifiable set of mathematical axioms.

All present or recent work in mathematical logic provides evidence for this fact, which is of a broadly inductive nature. What the past hundred-plus years have shown is that there are no irreducibly mathematical steps in argumentation. Mathematical experience teaches that mathematically valid arguments may be turned into logically valid arguments from a set of mathematical axioms relevant to the domain in question. For example, the truths of arithmetic follow logically from the axioms owed to Dedekind and named after Peano; those of analysis from the axioms characterising a complete ordered field; those of group theory from the axioms of group theory plus relevant supplementations in a given context (e.g., that a group is abelian or finite); likewise for truths of geometry, topology, set theory, and so on. All branches of mathematics have been axiomatised in such a way that a mathematically valid argument—a mathematical proof—may be cast as a logically valid argument from a set of mathematical axioms that applies to the branch (or branches) in question.

There remains room for disagreement about the logic in which mathematical arguments may all be cast. That is, about how to interpret the word ‘logical’ in the uncontroversial equation ‘mathematical validity = logical validity from mathematical axioms’. First-orderists take the relevant logic to be first-order logic: they believe that all mathematical proofs may be converted into first-order proofs from mathematical axioms. The conviction that this can always be done is what various authors, starting with Barwise (following a suggestion of Martin Davis), have called ‘Hilbert's Thesis’.[13] Naturally, to make this a substantive thesis, there must be some implicit constraints on which mathematical axioms are to be used. It will not do simply to add the conclusion as a premise every time. The axioms in a given branch must at least be held constant. It is difficult to spell out exactly what an appropriate axiomatisation is, but it is quite easy to recognise one in practice—just look at the countless formalisations logicians have produced of mathematical arguments.

As an illustration of Hilbert's Thesis, first-orderists contend that elementary set-theoretic arguments, such as $\mathcal{A}$, can all be turned into logically valid ones from the axioms of a first-order set theory (e.g., ZFC). More generally, those who think the correct foundational logic is compact (such as, but not limited to, first-orderists) will take the relevant logic to be compact. In contrast, second-orderists maintain that, at least in some instances, the relevant logic should be second-order logic. To capture all the truths of arithmetic, they argue, we must use second-order rather than first-order Peano Arithmetic. To capture those of analysis, we must use a second-order system (or subsystem) of analysis.[14] The mere statement of the axioms of topology requires second-order resources. And so on.

Suppose then that you take $\mathcal{A}$ to be mathematically but not logically valid. It follows from what we have said that $\mathcal{A}$ can be rendered logically valid by the addition of a mathematical premise or premises. Now this additional premise (or premises) must express in some way the idea that to be infinite is to be larger than any finite size. So let us assume that P is such a claim (or a collection of claims to this effect). The argument $\mathcal{A}$ supplemented with the required extra mathematical premise(s) becomes:

$$\begin{array}{c} \text{There is at least one planet.} \\ \text{There are at least two planets.} \\ \vdots \\ \text{There are at least } n \text{ planets.} \\ \vdots \\ P \\ \hline \text{There are infinitely many planets.} \end{array}$$

Call this augmented argument $\mathcal{A}^P$.

The problem now is that the non-compactness argument may be rerun on the supplemented argument $\mathcal{A}^P$. By assumption, $\mathcal{A}^P$ is logically valid. Yet no finite subset of $\mathcal{A}^P$'s augmented premise set implies its conclusion, since the claim (or claims) P together with finitely many claims of the form 'There are at least n planets' for finite n does not imply that there are infinitely many planets. We are still left with an argument witnessing English's non-compactness. This counterexample is not the original argument $\mathcal{A}$ but its supplementation $\mathcal{A}^P$.

To sum up the discussion: the objection we considered was that $\mathcal{A}$ is mathematically but not logically valid. Plainly, $\mathcal{A}$ is mathematically valid. Now, mathematical experience shows that mathematical validity is equatable to logical validity from appropriate mathematical axioms. But any supplementation of $\mathcal{A}$ by appropriate mathematical axioms will turn it into an argument that is logically valid but none of whose finite subarguments is valid. So we are still left with a logically valid argument exemplifying English's non-compactness.

4 | THE FINITE DESCRIPTION OBJECTION

The third line of resistance is that we cannot understand argument $\mathcal{A}$ except via some finite description. This description might be along the following lines:

> The argument $\mathcal{A}$ consists of the premises 'There are n planets' for each finite number n and the conclusion 'There are infinitely many planets'.

What's really going on when we understand $\mathcal{A}$, it is urged, is that we understand its finite description. To appreciate $\mathcal{A}$'s validity, we then reason in a finite way using this finite description. (Perhaps using set theory, arguing from the fact that, according to the argument, the set of planets is of size at least as great as any natural number to the conclusion that this set is of infinite size.) We thereby convince ourselves of $\mathcal{A}$'s validity using finite reasoning.

This third objection is a red herring. We grant its contention, which is that we understand $\mathcal{A}$ via a finite description and convince ourselves of $\mathcal{A}$'s validity by finite reasoning. Coming to appreciate $\mathcal{A}$'s validity is clearly not an infinitary undertaking. That does not change the fact that $\mathcal{A}$ is valid but that no finite subset of its premise set entails its conclusion. Arguments of English may consist of infinitely many premises even if we must employ finite terms to characterise them. $\mathcal{A}$'s status as a witness to English's non-compactness is unaffected.

Logic as we understand it aims to capture implicational facts. It is not a theory of what we understand when we grasp an argument's validity (or invalidity), or how we come to appreciate such facts. Hence our preference for the term 'implicational' over 'inferential': we are interested in what follows from what rather than what can be deduced from what.[15] Whether $\mathcal{A}$'s premises entail its conclusion is *not*, in other words, an anthropocentric question; it is a question about statements and the relations between them. The finite description objection thus confuses an epistemological fact (about how we convince ourselves of $\mathcal{A}$'s validity) with a logical one (about whether the relation of logical implication obtains between premises and conclusion).[16]

5 | THE INDETERMINACY OBJECTION

The final line of resistance queries whether natural language has a determinate consequence relation. According to this objection, there are only various consequence relations that arise from looking at English through a particular theoretical lens; none is the correct one. The above argument for the non-compactness of English assumes, falsely, that there is a determinate notion: the English consequence relation. Logical pluralists all take this line; indeed doing so seems tantamount to pluralism.[17]

Now logical pluralism has come under fire. We ourselves have given arguments against it and defended monism elsewhere.[18] But a more direct response is also available, which does not presuppose monism. Any account of natural-language consequence from the perspective of a particular logic $\mathscr{L}$ will yield a theoretical account of English consequence. In effect, such an account analyses or replaces the informal notion of consequence with the more precise notion of consequence$_{\mathscr{L}}$, the subscript indicating the way in which consequence is now being understood. For example, a first-orderist such as Quine would take the notion of consequence to be that of consequence$_{\mathrm{FOL}}$—consequence as modelled by FOL. Take, then, the notions of consequence$_{\mathscr{L}}$ and validity$_{\mathscr{L}}$ as informed by logic $\mathscr{L}$. Is $\mathcal{A}$ valid$_{\mathscr{L}}$ or not?[19] If yes, then on this conception English is non-compact, ruling out $\mathscr{L}$'s identification with FOL. If no, an explanation is owed as to why $\mathcal{A}$ appears valid despite its invalidity$_{\mathscr{L}}$. What form could such an explanation take?

In response, one could try to deploy one of the first three objections, in sections 2–4 above. But as we have seen, none of them succeeds. So we have yet to find a way to explain away the appearance of $\mathcal{A}$'s validity. To simply say that $\mathcal{A}$ is invalid because it cannot be formalised in $\mathscr{L}$ may be chalked up as a reason to expand one's foundational logic(s) beyond $\mathscr{L}$. In particular, to say that $\mathcal{A}$'s conclusion cannot be first-order formalised fails to explain why $\mathcal{A}$ appears valid; it merely highlights a shortcoming of first-order logic.

6 | UPSHOT

We have seen no good reason to doubt that the argument $\mathcal{A}$ (or some supplementation of it) witnesses English's non-compactness. Since only one counterexample is needed to refute a universal claim such as compactness, this shows that English is not compact. So no compact logic can fully capture English consequence. We were careful to argue for this conclusion without resting on any controversial assumptions about the nature of the logical constants. We assumed only that first-order logic is part of logic and did not appeal to criteria of logical constanthood such as topic-neutrality or generality. Our argument is consonant with such criteria, as developed by Tarski (1986) and Sher (1991) and in our own work (Griffiths & Paseau, forthcoming), which in particular support the logicality of the quantifier 'there are infinitely many'. But it does not presuppose them.

Our discussion rules out much more than just first-order logic. It affects a large class of logics, including any sound and complete one.[20] We must look to the non-compact logics instead.[21]

ENDNOTES

[1] Priest (2001) and Chapters 10 & 12 of Priest (2006). Priest contrasts this with logic's application to electronic circuitry, though even here one might think that what is being modelled are logical relations between propositions about circuits.

[2] Using standard or full semantics for second-order logic, under which it is well known to not be compact. When κ and λ are infinite cardinals, the logic $\mathscr{L}_{\kappa\lambda}$ extends first-order logic by allowing $< \kappa$ conjunctions and disjunctions of well-formed formulas, and existential or universal quantification over $< \lambda$ strings of variables; this defines a class of infinitary logics. It's easy to check that $\mathscr{L}_{\kappa\lambda}$ is not compact if κ is uncountable, e.g. by considering arguments with infinite premise sets whose conclusion is the conjunction of their premises.

[3] A finite subargument of a given argument is an argument with the same conclusion as the original argument's conclusion and whose premise set is a finite subset of the original argument's premise set.

[4] For the history of this emergence, see especially Moore (1980, 1988) and Shapiro (1991, ch. 7). In philosophy, first-order logic's greatest champion was Quine (1970).

[5] See Griffiths (2013) or chapter 1 of Griffiths and Paseau (forthcoming).

[6] Or 'one', 'one plus one', 'one plus one plus one', and so on. We could equally well use Hindu-Arabic numerals: 1, 2, 3, ..., 1,000,000, 1,000,001, ..., as they are part of English in our broad sense.

[7] Another champion of plural logic is Yi (2006, p. 262), who mentions another argument that is somewhat different from $\mathcal{A}$ but illustrates the same point. Yi's argument has premises interpretable as 'c_n is related to c_{n+1}', where each c_n (for n a natural number) is a name; its conclusion is interpretable roughly as 'There are some things such that for any one of them there is another one of them (which may be equal to the first) such that the first is related to the second'. Yi uses this example to demonstrate the non-compactness of plural logic and evidently regards its natural-language interpretation as logically valid. But he gives no further justification of the natural-language argument's logical validity beyond the fact that it is formalisable as a valid argument in plural logic.

[8] See Paseau (2019) and (forthcoming) for more discussion of this point.

[9] Perusal of virtually any work on the syntax or semantics of natural language supports this point. An early statement by Chomsky is: 'We might arbitrarily decree that such processes of sentence formation in English as those we are discussing cannot be carried out more than n times, for some fixed n. This would of course make English a finite state language, as, for example, would a limitation of English sentences to length of less than a million words. Such arbitrary limitations serve no useful purpose, however.' (Chomsky, 1957, p. 23). A more recent review article by Hauser, Chomsky and Fitch (2002, p. 1571) makes the same point: 'FLN [the Faculty of Language in the narrow sense] takes a finite set of elements and yields a potentially infinite array of discrete expressions. This capacity of FLN yields discrete infinity (a property that also characterises the natural numbers)…. the potential infiniteness of this system has been explicitly recognised by Galileo, Descartes, and the 17th-century "philosophical grammarians" and their successors, notably von Humboldt…. The core property of discrete infinity is intuitively familiar to every language user. Sentences are built up of discrete units: There are 6-word sentences and 7-word sentences, but no 6.5-word sentences. There is no longest sentence (any candidate sentence can be trumped by, for example, embedding it in "Mary thinks that…"), and there is no nonarbitrary upper bound to sentence length. In these respects, language is directly analogous to the natural numbers…' (2002, p. 1571). We thought this tendency universal until we happened on Ziff (1974) and Langendoen and Postal (1985). The former is a quixotic attempt by a philosopher to challenge the received view (as Ziff calls it) that English has infinitely many sentences. Langendoen and Postal (1985) also

challenge the thesis that natural languages have a countable infinity of sentences, but in the opposite direction. They contend that there are English sentences of *transfinite length*, because grammars that place no bounds on sentence length are simpler than those that do. So for Langendoen and Postal, the collection of English sentences is a proper class.

[10] That is, S does not logically entail s.

[11] Evidently, the word 'planet' could be replaced by any other common noun without affecting the argument's validity, so its validity does not turn on any specifically astronomical facts.

[12] The argument to follow can be extended to other forms of validity by considering the nesting of possible worlds. Much the usual picture is that the logically possible worlds form the broadest class, constrained only by logic. The mathematically possible worlds are a proper subset, constrained not only by logic but also by mathematics. The metaphysically possible worlds are in turn a proper subset of the mathematically possible ones, and thus a further proper subset of the logically possible ones. $\mathcal{A}$ is a *mathematically* valid argument, hence truth-preserving in all mathematically possible worlds. Since the mathematically possible worlds are a proper superset of the metaphysically possible worlds, it cannot be objected that $\mathcal{A}$ is *merely* metaphysically valid: it is valid in at least as strong a sense as *mathematical* validity. The same argument will extend to any notion of modality whose corresponding possible worlds are a subset of mathematically possible worlds.

[13] See Barwise (1977, p. 41).

[14] See for example the Reverse Mathematics programme outlined in Simpson (2010).

[15] See ch. 1 (esp. pp. 3–4) of Harman (1986) for a classic statement of the distinction between implication on the one hand and inference on the other.

[16] A related, non-epistemological, version of this objection is that the argument $\mathcal{A}$ is *grounded* in some way in a finite argument, e.g. an argument with a premise stating that for all n, 'There are at least n planets' is true, together with compositional truth principles. On a thumbnail, our response to such an objection is threefold. First, we are not aware of any worked-out and plausible accounts of what it is for some arguments to ground other arguments. Second, still less do we know of any such accounts whose conclusion is that all arguments are grounded by arguments in a compact logic. And third, if you believe in the idea of some arguments grounding others, you are very likely to think that arguments using semantic notions such as truth or semantic devices such as quotation are grounded in arguments that do not use them. Comparing $\mathcal{A}$ to an argument $\mathcal{B}$ whose premises include the claim that $\mathcal{A}$'s premises are true, $\mathcal{A}$ is the better candidate for the argument that does the grounding and $\mathcal{B}$ for the argument that is grounded.

[17] Beall and Restall (2006), reviewed in Paseau (2007), and Shapiro (2014) articulate and defend different varieties of pluralism.

[18] Part I of Griffiths and Paseau (forthcoming).

[19] We omit consideration of theoretical perspectives informed by $\mathscr{L}$ according to which $\mathcal{A}$ is neither valid$_{\mathscr{L}}$ nor invalid$_{\mathscr{L}}$, since this does not correspond to any live options in the literature. We also assume that the claim that no finite subset of $\mathcal{A}$'s premise set entails its conclusion is unimpeachable, whatever one's logical orientation.

[20] Since deductions are finite, any sound and complete logic, in which the deductive relation mirrors semantic consequence, must be compact.

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