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Quantum causality

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Traditionally, quantum theory assumes the existence of a fixed background causal structure. But if the laws of quantum mechanics are applied to the causal relations, then one could imagine situations in which the causal order of events is not always fixed, but is subject to quantum uncertainty. Such indefinite causal structures could make new quantum information processing tasks possible and provide methodological tools in quantum theories of gravity. Here, I review recent theoretical progress in this emerging area.

The concept of causality has been the subject of heated debates in literature about the metaphysics and philosophy of science for centuries. I have no intention of entering into these discussions here, but instead adopt a rudimentary, pragmatic approach. Causal thinking spontaneously arises in a child at about the time when she or he realizes that by exerting forces on nearby objects, the child can make these objects move according to their will. Causal relations are revealed by observing what would happen in the world (for instance, with the child's object) if a given parameter (the child's will) were separated out from the rest of the world and could be chosen freely.

The distinction between statistical and causal relations is echoed in the famous slogan "correlation does not imply causation". Whereas the former are definable in terms of joint probabilities for observed variables, the latter require specification of conditional probabilities to provide an analysis of how the probability distribution ought to change under external interventions[1]. Here, I will say that an A has a causal influence on B if conditional probability $P(B|A)$ for B observably changes under free variation of A. But how can we be sure that such a variation is 'really free'? We cannot, but this does not prevent us from considering a variation to be free whenever we have every reason to believe that it is. For all practical purposes, it is sufficient to toss a coin or use a quantum random generator to produce a free variable. And even if that 'free' variable were produced in a deterministic way — for example by taking the current temperature in Celsius, multiplying it by the number of my next-door neighbour's children plus one, I would still regard it as being 'free'.

In quantum physics, it is assumed that the background time or definite causal structure pre-exists such that for every pair of events A and B at distinct space–time regions one has either 'A is in the past of B', or 'B is in the past of A', or the two are space-like separated (see Fig. 1a,b). But thanks to the theorems developed by Kochen and Specker[2], and by Bell[3], we know that quantum mechanics is incompatible with the view that physical observables possess pre-existing values independent of the measurement context. (This incompatibility still holds if one assigns probabilities to the possible values of observables independently of the measurement context, rather than determining which particular result will be obtained in a single run of the experiment.) Do the theorems extend to causal structures as well?

If one assumes that quantum mechanical laws can be applied to causal relations, one might have situations in which the causal order of events is not always fixed, but is subject to quantum uncertainty, just like position or momentum. Indefinite causal structures could correspond to superpositions of situations where, roughly speaking, 'A is in the past of B' and 'B is in the past of A' jointly. One may speculate that such situations could arise when both general relativity and quantum physics become relevant. A simple example could involve a single massive object in a superposition of two or more distinct spatial positions. Because the object is in a spatial superposition, the gravitational field it produces will also be in a superposition of states, and so will the space–time geometry itself. This may lead to situations in which it is not fixed in advance whether a particular separation between two events is time-like or space-like.

The consequences of having indefinite causal order would be enormous, as this would imply that space–time and causal order might not be the truly basic ingredients of nature. It has been repeatedly pointed out that the notion of time might be at the origin of the persistent difficulties in formulating a quantum theory of gravity[4–9]. But how do we formulate quantum theory without the assumption of an underlying causal structure and background time? What new phenomenology would be implied by the idea of indefinite causal structures? If such structures cannot be excluded on a logical basis, do they exist in nature? And if they do, why have they not yet been observed in quantum experiments?

In 2005, Lucien Hardy proposed to address these penetrating questions by developing frameworks in which causal structures may be considered to be dynamic, as in general relativity, and indefinite, similar to quantum observables[10,11]. He introduced one such framework based on a new mathematical object, the 'causaloid', which contains information about the causal relations between different space–time regions. Since then, researchers, particularly in Pavia[12], Vienna[13] and the Perimeter Institute[14], have applied the powerful tools and concepts of quantum information to shed new light on the relation between the nature of time, causality and the formalism of quantum theory — a subject that has been traditionally studied within the general relativity and quantum gravity communities. In a similar vein, recent rigorous theorems in quantum information have been developed, which relate the probabilistic structure of quantum theory to the three-dimensionality of space[15,16]. By making plausible assumptions on how (microscopic) systems are manipulated by (macroscopic) laboratory devices, it was shown that the structure of the underlying probabilistic theory cannot be modified (for example by replacing quantum theory with a more general probabilistic theory) without changing the dimensionality of space.

In conventional (causal) formulation of quantum theory, correlations between results obtained in causally related and acausally related experiments are mathematically described in very different ways. For example, correlations between results obtained on a pair of space-like separated systems are described by a joint state on the tensor product of two Hilbert spaces, whereas those obtained from measuring a single system at different times are described by an initial state and a map on a single Hilbert space. (The causal structure of quantum theory is unrelated *a priori* to the causal structure of

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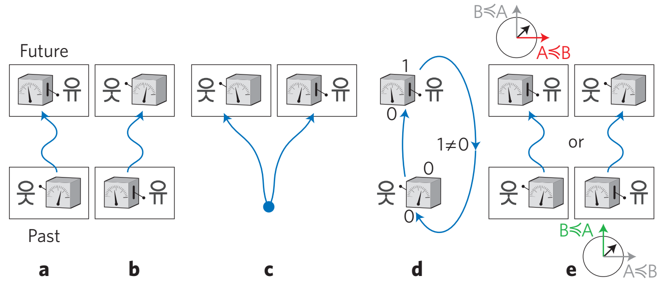

Figure 1 | Different causal relations between events in Alice's and Bob's laboratories. In a definite causal structure, a global background time determines whether **a**, Alice is before Bob, **b**, Bob is before Alice, or **c**, the two are causally neutral. Whereas in **a** and **b** signalling is always one-way, from the past to the future, there is no signalling in **c** because the two laboratories are space-like separated. The latter is a typical situation in tests of Bell's inequalities on entangled states. **d**, In a closed time-like causal structure, the signalling is two-way, which gives rise to the grandfather paradox. To illustrate this paradox, consider the following example. Alice performs an identity operation on her input bit of value 0. The unchanged bit leaves her laboratory and is sent to Bob, who performs a bit flip and outputs a bit of value 1. The bit travels 'backwards in time' to enter Alice's laboratory as her input. Hence, the logical contradiction $1 \neq 0$ for the value of Alice's input arises. This can be seen as an instance of the grandfather paradox if the bit values 1 and 0 are taken to represent 'killing Alice's grandfather' and 'not killing Alice's grandfather', respectively. **e**, In an indefinite causal structure, Bob can, by choosing his measurement basis, end up 'before' or 'after' Alice with a certain probability. The vector on the circle next to Bob's laboratory represents a resource — a 'process' — which gives rise to quantum correlations with indefinite causal order. If he performs a measurement in the 'red' ('green') basis, he 'projects' the process such that his actions occur after (before) Alice's operations.

relativity, as, for example, the measurements on two systems may be 'time-like' separated in the relativistic sense and still be described by a tensor product of Hilbert spaces. Here, for simplicity, I will use 'causally related' and 'time-like', as well as 'acausally related' and 'space-like', interchangeably.) Very much at the focus of recent research on causality in quantum theory is the objective of finding a unified way of representing correlations between space-like and time-like regions. Once such a representation is found, we may be able to use it for the description of general quantum correlations, for which the causal ordering of events and whether they take place between space-like or time-like regions is not fixed.

Various results in the past have indicated that a unified quantum description may be possible for experiments involving distinct systems at one time and those involving a single system at distinct times. For example, it has been shown that there is an isomorphism between spatial and temporal quantum correlations[17–20]. This has conceptual and practical implications for the correspondence between quantum bounds on violation of the Bell inequality[21], and its temporal analogue, the Leggett–Garg inequality[22–25]. The correspondence between the communication costs in the classical simulation of spatial correlations and the memory costs in the simulation of temporal correlations is an example of this[17,26]. Eventually all these developments led to several approaches towards a causally neutral formulation of quantum theory: the causaloid[5] already mentioned above, and further developments in terms of duotensors[27], the quantum combs[28], quantum processes[13] and quantum conditional states[14].

Notwithstanding the differences among the approaches, they all make use of the Choi–Jamiolkowski (CJ) isomorphism[29,30] to provide a unified framework for representing a composition of operations as well as tensor products of system states. If an operation is performed on a quantum state described by a density matrix ρ, $\mathcal{M}(\rho)$ describes the updated state after the operation (up to normalization), where $\mathcal{M}$ is a completely positive (CP), trace-non-increasing map (because we want our maps to lead to positive probabilities not larger than one) from the space of matrices over the input Hilbert space A to the one over the output Hilbert space B, which we write as $\mathcal{M}: \mathcal{L}(A) \to \mathcal{L}(B)$ (the two Hilbert spaces can have different dimensions, as the operation may involve additional quantum systems). The CJ isomorphism enables us to represent the operations by operators rather than maps. It associates a bipartite state $M^{AB} \in \mathcal{L}(A \otimes B)$ to a CP map, as given by $M^{AB} = \mathcal{J} \otimes \mathcal{M}(|\phi^+\rangle\langle\phi^+|)$, where $\otimes$ indicates tensor product, $|\Phi^+\rangle = \Sigma_{j=1}^{d_A}|jj\rangle \in A \otimes B$ is a (non-normalized) maximally entangled state, the set of states $\{|j\rangle\}_{j=1}^{d_A}$ is an orthonormal basis of A with dimension d_A, and $\mathcal{J}$ is the identity map.

In the comb[28] and duotensor[27] framework, one associates CJ operators with arbitrary regions of space–time in which an observer might possibly perform a quantum operation. These operators can be combined to obtain the operator for a bigger, composite, region, using methods that are motivated in part by the graphical representation of categorical quantum mechanics[31]. In the framework of quantum conditional states, Matt Leifer and Rob Spekkens[14] have developed a causally neutral formulation of quantum theory using a quantum generalization of Bayesian conditioning[32]. They introduce a conditional state $\rho_{B|A}$, playing an analogous role to conditional probability $P(B|A)$ in classical probability theory. If A and B are space-like separated regions, their joint state ρ_{AB} is inferred from the conventional formalism, and the conditional state is derived from the joint, whereas if they are time-like separated it is the conditional state $\rho_{B|A}$ that is inferred from the conventional formalism (for example through a map $\rho_{B|A} = \mathcal{M}(\rho_A)$), and the joint state is derived. In either case the relation between the conditional and joint state is given by $\rho_{AB} = \rho_{B|A} \star \rho_A$ where the $\star$-product is a particular product (defined by $\rho_{B|A} \star \rho_A \equiv \sqrt{\rho_A}\, \rho_{B|A} \sqrt{\rho_A}$, where I have dropped the identity operators and tensor products), which is analogous to the Bayes relation in classical probability theory, $P(AB) = P(B|A)P(A)$. But the approach has limitations, for example in treating multiple temporal correlations, mostly owing to the fact that the $\star$-product is non-commutative and non-associative. (The approaches[12–14,27,28] differ among themselves in the insertion of partial transposes in the definition of CJ operators.)

With the notable exception of ref. 14, which has an epistemological flavour, all other approaches are typically formulated operationally; instead of using the notions of 'traditional' physics such as position, momentum or energy, the focus lies on instrument settings and the outcomes of measurements. The operational idea of a causal influence is best illustrated by considering two scientists, Alice and Bob, who work in two separate laboratories. At every run of the experiment, each of them receives a physical system and performs an operation on it, after which they send their respective system out of the laboratory. During the operations of each experimenter, the laboratory is shielded from the rest of the world — it is only opened for the system to come in and to go out, but except for these two events, it is kept closed and a signal can neither enter into nor leak out of the laboratory. Each laboratory features a device with an input and an output connector. If input a is chosen on Alice's side (or, respectively, b on Bob's side), she will perform an operation on the system and send it out of the laboratory. The device will output measurement result x (respectively y) according to a certain probability distribution $p(x,y|a,b)$. The operations a and b, for example, could be the flip of a classical bit in the classical world or the unitary transformation, or in general a CP trace-non-increasing map in the quantum world.

The correlations are non-signalling if no observable change can take place in Alice's laboratory as a consequence of anything that may be done in Bob's laboratory and vice versa. More specifically, this implies that Alice's marginal distribution — which is obtained by summing up the joint probability distribution over Bob's results — is independent of Bob's input choice and vice versa: $\sum_y p(x,y|a,b) = p(x,a)$ and $\sum_x p(x,y|a,b) = p(y,b)$ for all a, b, x and y. The correlations are one-way signalling if one of the two conditions does not hold, and two-way signalling if neither condition holds.

It can easily be seen that a fixed causal order will impose restrictions on the ways in which Alice and Bob can communicate. Imagine that Alice exists in Bob's past. She can act on her system and encode her input a into it before sending it to Bob. That way, his device can output $y = a$ and the signalling is perfect. Because each party receives each system only once, Bob cannot signal to Alice. Consequently, two-way signalling is impossible. I will denote by $p^{A\preccurlyeq B}(x,y|a,b)$ (by $p^{B\preccurlyeq A}(x,y|a,b)$) the general probability distribution in which signalling from Alice to Bob (from Bob to Alice) is possible. In a definite causal structure, it may still be the case that the causal relation between events is not known with certainty. A situation where Alice exists before Bob with a probability of $0 \leq \lambda \leq 1$ and Bob exists before Alice with a probability of $1 - \lambda$ is represented by a probability of the form of $p(x,y|a,b) = \lambda p^{A\preccurlyeq B}(x,y|a,b) + (1 - \lambda)\, p^{B\preccurlyeq A}(x,y|a,b)$.

Are more general causal structures possible? Can Alice and Bob have two-way communication even though the exchanged system enters each of the two laboratories only once? At first sight this seems impossible, except in a world with closed causal loops, where a signal may go back and forth from Alice to Bob. Such closed time-like curves (CTCs) were first proposed by Kurt Gödel in 1949. Gödel was an Austrian logician who discovered, surprisingly, that general relativity equations allow CTC solutions[33]. But the existence of CTCs seems to imply logical paradoxes, most notably the 'grandfather paradox' in which an agent goes back in time to kill his grandfather (see Fig. 1d). Possible solutions have been proposed in which quantum mechanics and CTCs can coexist and such paradoxes are avoided, but not without modifying quantum theory into a nonlinear one (refs 34–37, and unpublished results by C. H. Bennett and B. Schumacher). Nonlinear theories themselves are problematic[38]. The question remains: is it possible to keep the (linear) framework of quantum theory, have no paradoxes and still go beyond definite causal structures?

One such framework was proposed recently by Ognyan Oreshkov, Fabio Costa and I[13]. There, we assumed that operations in local laboratories are described by quantum mechanics (that is, they are CP maps). Using the CJ isomorphism, the probability for a pair $M_{x,a}^{A_1A_2}$ and $M_{y,b}^{B_1B_2}$ of local CP maps performed by Alice between the local input A_1 and output A_2 Hilbert spaces, and by Bob between the Hilbert spaces B_1 and B_2, are represented as a bilinear function of the corresponding CJ operators as follows: $p(x,y|a,b) = \mathrm{Tr}[W^{A_1B_1A_2B_2}(M_{x,a}^{A_1A_2} \otimes M_{y,b}^{B_1B_2})]$. Here $W^{A_1B_1A_2B_2}$ belongs to the space of matrices over the tensor product of the input A_1,B_1 and the output A_2,B_2 Hilbert spaces of two parties. It is the central object of the formalism and represents a new resource called 'process' — a generalization of the notion of 'state'. The matrix W is a positive matrix, and it returns unit probability for CP trace-preserving maps. Just like in the aforementioned approaches, it provides a unified way to represent correlations in casually related and acausally related experiments. Although the notion of causal structure is built within the local laboratories insofar as the output of an operation is causally influenced by the input, no reference is made to any global causal relations between the operations in two laboratories. Most interestingly, we have found situations where two operations are neither causally ordered nor in a probabilistic mixture of definite causal orders: that is, one cannot say that one operation is either before or after the other. In these cases the process is not a probabilistic mixture of the processes with definite causal order: $W^{AB} \neq \lambda W^{A\preccurlyeq B} + (1 + \lambda) W^{B\preccurlyeq A}$, where $0 \leq \lambda \leq 1$, and $W^{A\preccurlyeq B}$ is the process in which Alice can signal to Bob, and $W^{B\preccurlyeq A}$ is that in which Bob can signal to Alice.

In terms of probability distributions, this can be written as $p(a,b|x,y) \neq \lambda p^{A\preccurlyeq B}(a,b|x,y) + (1 - \lambda)\, p^{B\preccurlyeq A}(a,b|x,y)$. Because the correlations are incompatible with any underlying definite causal structure, we call them 'quantum correlations with indefinite causal order'.

The existence of the new correlations can be demonstrated in a theory-independent way on the basis of recorded data in an experimental test. These new correlations violate a 'causal inequality' which is satisfied if events take place in a causal sequence. This stands in direct analogy to the famous violation of Bell's inequality in quantum mechanics, which is satisfied if the measured quantities have predefined local values[3]. The 'causal inequality' is best explained in terms of a game involving two players, Alice and Bob again, each of whom receives a random input bit value, 0 or 1. The point of the game is that each player tries to guess the input of the other player. One of the players, say Bob, receives an additional random bit, which specifies who will need to guess whose bit in a given run of the game. It can easily be seen that in every causal scenario the success probability of the game is bounded by 3/4. Without loss of generality, consider that Alice is in Bob's past, as illustrated in Fig. 1a. Then she can always send her input bit to him and they will accomplish their task perfectly if he is required to guess her bit, whereas if she is asked to guess his bit, she cannot do better than giving a random answer. This gives an overall success probability of 3/4. But if Alice and Bob share quantum correlations with indefinite causal order, they can achieve a success probability as high as[13] $(2 + \sqrt{2})/4$. Whereas causal correlations allow signalling in no more than one fixed direction, correlations with indefinite causal order allow Bob, depending on his choice of measured observable, to effectively end up 'before' or 'after' Alice with a probability of $1/\sqrt{2}$ (see Fig. 1e). All causal loops and paradoxes are avoided: in every single run, only one-way signalling is realized, but the signalling direction, from Alice to Bob or from Bob to Alice, is in Bob's control and may vary from run to run. It is intriguing that both the classical bound and the quantum violation of the causal inequality match the corresponding numbers in the Clauser–Horne–Shimony–Holt version[39] of Bell's inequality. Most recently, a process for three parties has been found in which perfect signalling correlations among three parties are possible, whereas the same is impossible in any causal scenario[40] (this is analogous to the 'all versus nothing' type of argument against local hidden variables[41]).

The possibility of indefinite causal orders has also been discussed in the context of quantum computation[42]. The idea of (causal) quantum computation, or quantum circuits, may be illustrated as a set of gates physically connected by 'wires' through which quantum systems propagate. As the systems pass those gates, they change their states. This is repeated in succession until the computation is

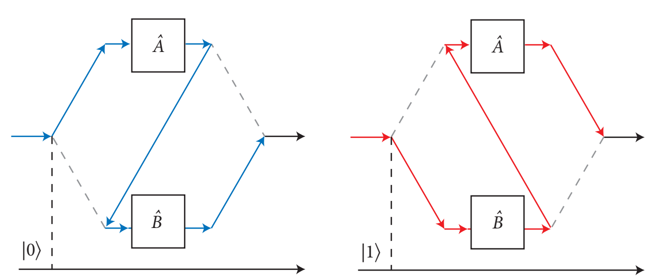

Figure 2 | Superposition of quantum circuits. The causal succession in which the physical boxes $\hat{A}$ and $\hat{B}$ are applied to the computer's state depends on the state of the control qubit. By projecting the control qubit in a particular basis ($\frac{1}{\sqrt{2}}(|0\rangle \mp |1\rangle)$; see text for more details), the network is in a quantum superposition of being used in a circuit with causal structure $A \preccurlyeq B$ and of being used in a circuit with causal structure $B \preccurlyeq A$.

finished. Once two gates have been inserted in a circuit in a given order, there is no way to invert their causal relation.

Recently, Chiribella *et al.* have proposed that the geometry of the wires between the gates could be controlled by the quantum state of a controlled qubit, a 'quantum switch'[12]. In this way it would be possible to build a superposition of circuits in which the gates of a given set act in a different order, depending on the state of the switch (Fig. 2). This more general model of computation can outperform 'causal' quantum computers in specific tasks, such as discriminating between two non-signalling channels[43]. In a simpler version, the task is to distinguish whether a pair of boxes — which represent two unitaries $\hat{A}$ and $\hat{B}$ — commute or anticommute, that is, whether $\hat{A}\hat{B} = \pm\, \hat{B}\hat{A}$. Realizing such a task within the standard circuit model would unavoidably require at least one of the unitaries to be applied twice[12]. But there is a simple algorithm that exploits superpositions of causal circuits and makes it possible to use each box only once. It coherently applies the two unitaries on the initial state $|\psi\rangle$ of the computer (for example an internal degree of freedom of the flying qubit) in two possible orders, depending on the state of a control qubit. If the control qubit is prepared in the superposition $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, the output of the algorithm is: $\frac{1}{\sqrt{2}}(\hat{A}\hat{B}|\psi\rangle|0\rangle + \hat{B}\hat{A}|\psi\rangle|1\rangle) = \frac{1}{\sqrt{2}}\hat{A}\hat{B}|\psi\rangle(|0\rangle \pm |1\rangle)$ with the phase of the control qubit state +1 or −1 depending on whether the two unitaries commute or anticommute, respectively. A simple measurement of the control qubit in the bases $\frac{1}{\sqrt{2}}(|0\rangle \mp |1\rangle)$ finally solves the task. In this elementary example, the number of oracle queries is reduced from 2 to 1 with respect to causal computation, but superpositions of quantum circuits can be exploited to solve a related computational problem of size n by using $O(n)$ black-box queries, whereas the best-known quantum algorithm with fixed order between the gates requires $O(n^2)$ queries[44]. In general, any superposition of quantum circuits can be simulated (with, at most, polynomial overhead) by a standard causal quantum circuit.

The causal succession of gate operations in a quantum circuit is the circuit analogue of the events separated by a time-like interval. This analogy can be pushed further. The networks in which geometry of the wires between the gates are entangled with the state of a control qubit can be thought of as a toy model for the quantum gravity situation introduced in the text above. If a massive object is put in a spatial superposition, then the metric produced, and hence spatio-temporal distances between events in the gravitational field of the object, get entangled with the object's position.

We have only begun to scratch the surface of this new field of quantum causality. The deeper we dig, the more questions arise. Where should we search for phenomenological evidence of indefinite causal orders? At the Planck length? In superpositions of large masses and space–time backgrounds? Although it is evident how to implement superpositions of wires in a circuit, they are apparently not sufficient for realizing indefinite causal processes that violate the causal inequality. Are these processes just mathematical artefacts of the theory? What is the 'quantum bound' on violating the causal inequality, and which physical principles might constrain it? What is the zoo of complexity classes for quantum computers exploiting indefinite causal structures? If space–time and causality are not the most fundamental ingredients of nature, what, then, are the basic building blocks? And might the former emerge in some sort of 'decoherence' from the latter?

In the same way that quantum entanglement and coherence as working concepts have given rise to quantum-enhanced information processing[45], for instance with Shor's factoring algorithms or secure quantum key distribution, the power of quantum computation on indefinite causal structures may lead to new protocols and procedures, maybe even changing the character of quantum information science itself. But the present research programme will not reach fulfilment if it does not provide new insights into the challenge of finding a theory of quantum gravity. The difficulties that arise when attempting to merge quantum theory and general relativity are so complex, and have lasted for so long, that some have come to suspect that they are not merely technical and mathematical in nature, but rather conceptual and fundamental. Research on quantum causality may lead to answers.

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Competing financial interests

The authors declare no competing financial interests.